

## 05. Symmetrical Components & Fault Calculations

⇒ 3-phase System :- Any three coplanar vectors  $v_a$ ,  $v_b$ , and  $v_c$  can be expressed in terms of three new vectors  $v_1$ ,  $v_2$  and  $v_3$  by three simultaneous linear equation with constant coefficients. Thus

$$v_a = a_{11}v_1 + a_{12}v_2 + a_{13}v_3$$

$$v_b = a_{21}v_1 + a_{22}v_2 + a_{23}v_3$$

$$v_c = a_{31}v_1 + a_{32}v_2 + a_{33}v_3$$

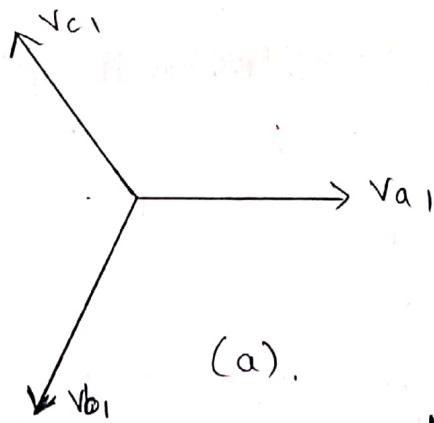
Each of the original vectors has been replaced by a set of three vectors making a total of nine vectors. This has been done to simplify the calculation and to have better understanding of the problem. With this in mind, two conditions should be satisfied in selecting systems of components to replace 3-phase current and voltage vector:

1. Calculation should be simplified by the use of the chosen system of components. This is possible only if the impedances (or admittance) associated with the components of current (or) voltage can be obtained readily by calculation or test.

2. The system of components chosen should have physical significance and be an aid in determining power system performance.

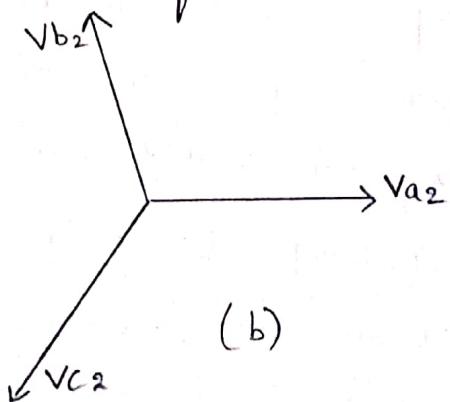
According to the Fortescue's theorem, the three unbalanced vectors  $V_a$ ,  $V_b$  and  $V_c$  can be replaced by a set of three balanced system of vectors. Therefore, the solution of equations is unique. A balanced system of three vectors is one in which the vectors are equal in magnitude and are equi-spaced. The three symmetrical components replacing  $V_a$ ,  $V_b$  and  $V_c$  are.

1. positive sequence component which has three vectors of equal magnitude but displaced in phase from each other by  $120^\circ$  and has the same phase sequence as the original vectors.
2. negative sequence component which has three vectors of equal magnitude but displaced in phase from each other by  $120^\circ$  and has the phase sequence opposite to the original vectors.
3. Zero sequence component which has three vectors of equal magnitude and also are in phase with each other.



(a).

(a) positive sequence component.



(b)

(b) negative sequence component.

$$\begin{array}{l} \longrightarrow V_{a0} \\ \longrightarrow V_{b0} \\ \longrightarrow V_{c0} \end{array}$$

### (C) Zero Sequence component.

The components has been shown in. the voltage vectors has been designated as  $V_a$ ,  $V_b$  and  $V_c$  and the phase sequence is assumed here as a,b,c the subscripts 1,2 and 0 and being used to represent positive, negative and zero sequence quantities respectively.

$\Rightarrow$  Significance of positive, negative and zero sequence

components:- By a positive Sequence system of vectors is meant the vectors are equal in magnitude and  $120^\circ$  apart in phase, in which the time order of arrival of the phase vectors at a fixed axis of reference corresponding to the generated voltages. This Really means that if a set of positive sequence voltages is applied to the stator winding of the alternator, the direction of rotation of the stator field is the same as the Rotor, the set of voltage are positive sequence voltages. on the contrary, if the direction of rotation of the stator field is opposite to that of the Rotor, the set of voltages are negative sequence voltages. The zero sequences of voltages are single phase voltage and, therefore, they give rise to an alternating field in space. Since the 3-phase windings are  $120^\circ$  apart in space, at any particular instant the three phases are  $120^\circ$  apart and, vector fields due to the three phases are  $120^\circ$  apart and, therefore, assuming complete symmetry of the windings the net flux in the air gap will be zero.

from the following Relations between the original unbalanced vectors and their corresponding symmetrical components, can be written:

$$V_a = V_{a1} + V_{a2} + V_{a0}$$

$$V_b = V_{b1} + V_{b2} + V_{b0}$$

$$V_c = V_{c1} + V_{c2} + V_{c0}$$

Assuming phase a as the reference as the following relations between the symmetrical components of phases b and c in terms of phase a can be written. Here use is made of the operator  $\lambda$  which has a magnitude of the unity and rotation through  $120^\circ$ , i.e. when any vector is multiplied by  $\lambda$ , the vector magnitude remains same but is rotated anticlockwise through  $120^\circ$ . Thus.

$$\lambda = 1 \angle 120^\circ$$

In the complex form  $\lambda = \cos 120^\circ + j \sin 120^\circ$

Similarly.  $\lambda^2 = -0.5 + j0.866$

$$\lambda^3 = 1.0 = 1 \angle 360^\circ$$

$$\lambda^3 - 1 = 0$$

$$(\lambda - 1)(\lambda^2 + \lambda + 1) = 0$$

Since  $\lambda \neq 1$  as  $\lambda$  is a complex quantity as defined above.

$$\therefore \lambda^2 + \lambda + 1 = 0$$

In fact  $\lambda$  is a number which when doubly squared remains  $\lambda$  itself, i.e.  $\lambda^4 = \lambda$ .

So the important relations that will be frequently required in power system analyses are.

$$\lambda = -0.5 + j0.866 = 1.0 \angle 120^\circ$$

$$\lambda^2 = -0.5 - j0.866 = 1.0 \angle 120^\circ$$

$$\lambda^3 = 1.0 < 0$$

$$\lambda^4 = \lambda$$

$$\lambda^2 + \lambda + 1 = 0$$

now we go back to deriving relations between the symmetrical components of phases b and c in terms of the symmetrical components of phase a.

$$V_{b1} = \lambda^2 V_{a1}$$

This means in order to express  $V_{b1}$  in terms of  $V_{a1}$ ,  $V_{a1}$  should be rotated anti-clockwise through  $240^\circ$ .

similarly  $V_{c1} = \lambda V_{a1}$

for negative sequence vector.

$$V_{b1} = \lambda V_{a2}, V_{c2} = \lambda^2 V_{a2}$$

for zero sequence vectors.

$$V_{b0} = V_{a0} = V_{c0}$$

Substituting these relations in equations

$$V_a = V_{a1} + V_{a2} + V_{a0}$$

$$V_b = \lambda^2 V_{a1} + \lambda V_{a2} + V_{a0}$$

$$V_c = \lambda V_{a1} + \lambda^2 V_{a2} + V_{a0}$$

Comparing equations with equations.

$$a_{11} = a_{12} = a_{13} = 1$$

$$a_{21} = \lambda^2, a_{22} = \lambda, a_{23} = 1$$

$$a_{31} = \lambda, a_{32} = \lambda^2, a_{33} = 1$$

The co-efficients have been uniquely determined for the 3-phase systems. Equations express the phase voltages  $V_a$ ,  $V_b$  and  $V_c$  in terms of the symmetrical components of phase a, in case  $V_{a1}$ ,  $V_{a2}$  and  $V_{a0}$  are known, the phase voltages  $V_a$ ,  $V_b$  and  $V_c$  can be calculated.

(3)

Similar relations between the phase currents in terms of the symmetrical components of currents taking phase  $a$  as reference hold good and given below.

$$I_a = I_{a1} + I_{a2} + I_{a0}$$

$$I_b = \lambda^2 I_{a1} + \lambda I_{a2} + I_{a0}$$

$$I_c = \lambda I_{a1} + \lambda^2 I_{a2} + I_{a0}$$

Normally the unbalanced phase voltages and currents are known in a system; it is required to find out the symmetrical components. The procedure is as follows.

The problem is given  $V_a, V_b, V_c$  find out  $V_{a1}, V_{a2}$  and  $V_{a0}$ . To find out positive sequence component  $V_{a1}$ , multiply equation by 1,  $\lambda$  and  $\lambda^2$  respectively and adding them up, it gives.

$$\begin{aligned} V_a + \lambda V_b + \lambda^2 V_c &= V_{a1}(1 + \lambda^3 + \lambda^3) + V_{a2}(1 + \lambda^2 + \lambda^4) \\ &\quad + V_{a0}(1 + \lambda + \lambda^2) \\ &= 3V_{a1} + V_{a2}(1 + \lambda^2 + \lambda) + 0 \\ &= 3V_{a1} \end{aligned}$$

Since

$$1 + \lambda + \lambda^2 = 0$$

$\therefore V_{a1} = \frac{1}{3} [V_a + \lambda V_b + \lambda^2 V_c]$   
For negative sequence component  $V_{a2}$  multiplying equations and by 1,  $\lambda^2$  and  $\lambda$  respectively and adding.

$$\begin{aligned} V_a + \lambda^2 V_b + \lambda V_c &= V_{a1}(1 + \lambda^2 + \lambda^2) + V_{a2}(1 + \lambda^3 + \lambda^3) \\ &\quad + V_{a0}(1 + \lambda^2 + \lambda) \\ &= 3V_{a2} \end{aligned}$$

$$V_{a2} = \frac{1}{3} [V_a + \lambda^2 V_b + \lambda V_c]$$

For zero sequence component  $V_{a0}$ , add equations.

$$V_a + V_b + V_c = V_{a1}(1 + \lambda^2 + \lambda) + V_{a2}(1 + \lambda + \lambda^2) + 3V_{a0}$$

$$V_{a0} = \frac{1}{3} (V_a + V_b + V_c)$$

Rewriting these equations,

$$V_{a1} = \frac{1}{3}(V_a + \lambda V_b + \lambda^2 V_c)$$

$$V_{a2} = \frac{1}{3}(V_a + \lambda^2 V_b + \lambda V_c)$$

$$V_{ao} = \frac{1}{3}(V_a + V_b + V_c)$$

Similarly these relations for current are given as-

$$I_{a1} = \frac{1}{3}(I_a + \lambda I_b + \lambda^2 I_c)$$

$$I_{a2} = \frac{1}{3}(I_a + \lambda^2 I_b + \lambda I_c)$$

$$I_{ao} = \frac{1}{3}(I_a + I_b + I_c)$$

$\Rightarrow$  Average 3-phase power in terms of symmetrical components:-

The average power.

$$\begin{aligned} P &= V_a I_a \cos \phi_a + V_b I_b \cos \phi_b + V_c I_c \cos \phi_c \\ &= V_a \cdot I_a + V_b \cdot I_b + V_c \cdot I_c \\ &= (V_{a1} + V_{a2} + V_{ao}) \cdot (I_{a1} + I_{a2} + I_{ao}) \\ &\quad + (\lambda^2 V_{a1} + \lambda V_{a2} + V_{ao}) \cdot (\lambda^2 I_{a1} + \lambda I_{a2} \\ &\quad + I_{ao}) \\ &\quad + (\lambda V_{a1} + \lambda^2 V_{a2} + V_{ao}) \cdot (\lambda I_{a1} + \lambda^2 I_{a2} + I_{ao}) \end{aligned}$$

Taking first term on the r.h.s

$$\begin{aligned} &(V_{a1} + V_{a2} + V_{ao}) \cdot (I_{a1} + I_{a2} + I_{ao}) \\ &= V_{a1} \cdot I_{a1} + V_{a2} \cdot I_{a2} + V_{ao} \cdot I_{ao} + V_{a1} \cdot I_{a2} + V_{a1} \cdot I_{ao} \\ &\quad + V_{a2} \cdot I_{a1} + V_{ao} \cdot I_{a1} + V_{ao} \cdot I_{a2} \end{aligned}$$

Expanding second term on the r.h.s

$$\begin{aligned} &(\lambda^2 V_{a1} + \lambda V_{a2} + V_{ao}) \cdot (\lambda^2 I_{a1} + \lambda I_{a2} + I_{ao}) \\ &= \lambda^2 V_{a1} \cdot \lambda^2 I_{a1} + \lambda^2 V_{a1} \cdot \lambda I_{a2} + \lambda^2 V_{a1} \cdot I_{ao} + \lambda V_{a2} \cdot \lambda^2 I_{a1} \\ &\quad + \lambda V_{a2} \cdot \lambda I_{a2} + \lambda V_{a2} \cdot I_{ao} + V_{ao} \cdot \lambda^2 I_{a1} + V_{ao} \cdot \lambda I_{a2} \\ &\quad + V_{ao} \cdot I_{ao} \end{aligned}$$

now the dot product of two vectors does not change when both are rotated through the same angle.

For example,

$$\lambda^2 V_{a1} \cdot \lambda^2 I_{a1} = V_{a1} \cdot I_{a1}$$

$$\lambda^2 V_{a1} \cdot \lambda I_{a2} = \lambda V_{a1} \cdot I_{a2}$$

The addition of the terms after expanding and Rearranging.

$$P = 3V_{a0} \cdot I_{a0} + 3V_{a2} \cdot I_{a2} + 3V_{a1} \cdot I_{a1} + V_{a1} \cdot I_{a2} (1+\lambda+\lambda^2)$$

$$+ V_{a1} \cdot I_{a1} (1+\lambda+\lambda^2) + V_{a2} \cdot I_{a1} (1+\lambda+\lambda^2) + V_{a2} \cdot I_{a0}$$

$$+ V_{a0} \cdot I_{a1} (1+\lambda+\lambda^2) + V_{a0} \cdot I_{a2} (1+\lambda+\lambda^2)$$

$$= 3(V_{a1} \cdot I_{a1} + V_{a2} \cdot I_{a2} + V_{a0} \cdot I_{a0})$$

$$= 3[V_{a1}|I_{a1}| \cos\phi_1 + |V_{a2}| |I_{a2}| \cos\phi_2 + |V_{a0}| |I_{a0}| \cos\phi_0]$$

The same power expression can be very easily derived using matrix manipulations.

$$P+jQ = V_a I_a^* + V_b I_b^* + V_c I_c^*$$

$$= \begin{bmatrix} V_a & V_b & V_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$

Since from equations,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = AV$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T = (AV)^T = V^T A^T$$

$$P+jQ = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \begin{bmatrix} V_{a0} & V_{a1} & V_{a2} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$

Now substituting for the phase current the corresponding symmetrical components noting that  $\lambda$  and  $\lambda^2$  are conjugate.

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

$$P+jQ = \begin{bmatrix} V_{a0} & V_{a1} & V_{a2} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

$$= \begin{bmatrix} V_{a0} & V_{a1} & V_{a2} \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

$$\begin{aligned}
 &= 3 \begin{bmatrix} V_{a0} & V_{a1} & V_{a2} \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \\
 &= 3 \left[ V_{a0} I_{a0}^* + V_{a1} I_{a1}^* + V_{a2} I_{a2}^* \right] \\
 P &= 3 \left[ |V_{a0}| |I_{a0}| \cos\phi_0 + |V_{a1}| |I_{a1}| \cos\phi_1 + |V_{a2}| |I_{a2}| \cos\phi_2 \right].
 \end{aligned}$$

$\Rightarrow$  Sequence Impedance:-

The positive sequence impedance of an equipment is the impedance offered by the sequence impedance of the equipment to the flow of positive sequence currents. Generally, the negative sequence or zero sequence impedance of the equipment is the impedance offered by the equipment to the flow of corresponding sequence current.

Let us represent positive, negative and zero sequence impedance respectively by  $Z_1$ ,  $Z_2$  and  $Z_0$ . We have already mentioned that for the symmetrical system there is no mutual coupling between the sequence networks. The three-sequence system can be then be considered separately and phase current and voltages determined by superposing their symmetrical components of current and voltage respectively.

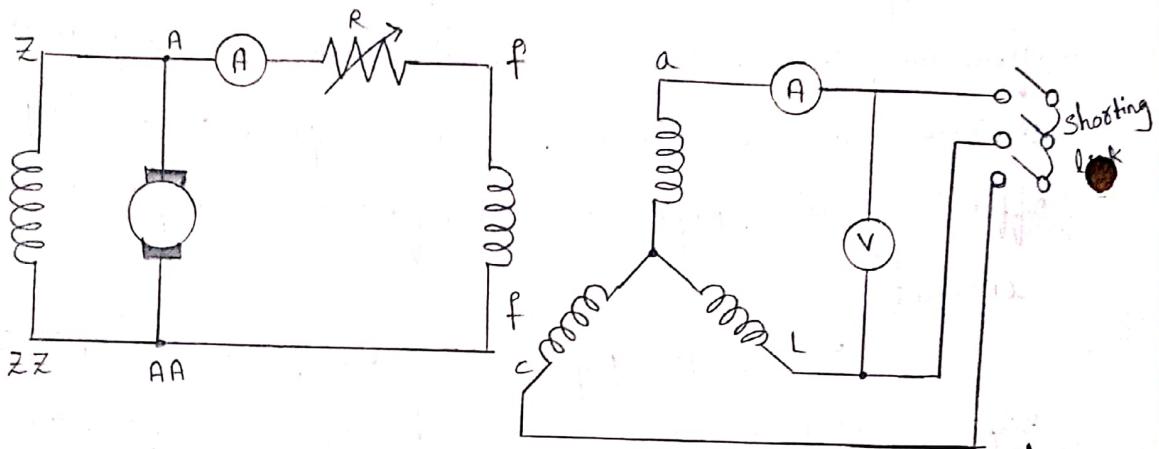
$\Rightarrow$  measurement of sequence Impedance of Rotating machines:-

Measurement of positive sequence impedance: As already mentioned the positive sequence impedance depends upon the working of the machine i.e. whether it is working under subtransient, transient or steady state condition. The impedance under steady state condition is known as the synchronous impedance and is measured by the well-known open circuit short circuit test. This impedance is defined as.

field current at rated armature const  
on sustained symmetrical short circuit.

Synchronous Impedance  $\text{en.p.u} = \frac{\text{Field current at normal open circuit}}{\text{Voltage on the air gap line (i.e. the extended straight line part of the magnetisation curve)}}.$

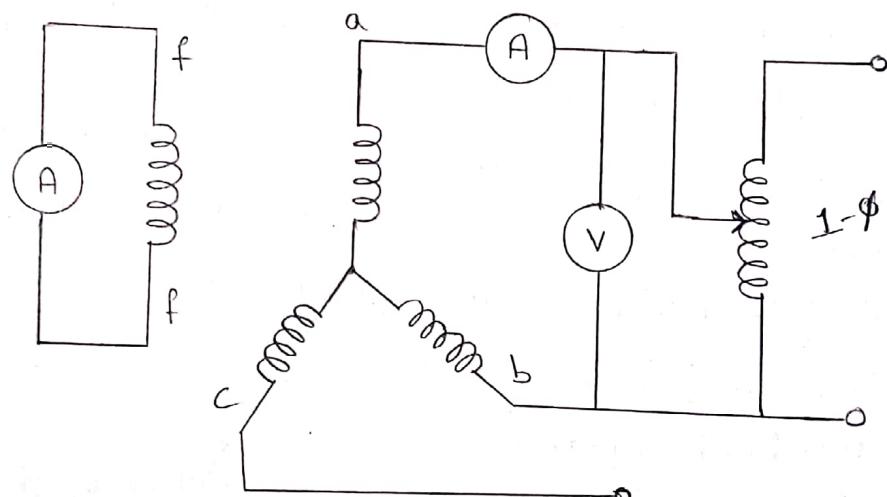
method of test for Synchronous Impedance: The machine is run at synchronous speed. In proper direction with the help of a pre-mover.



Connection diagram for open circuit and short circuit test on an alternator  
The switch is kept in off position to perform open circuit test  
The Readings of voltmeter for various field current are taken  
next the excitation is reduced to minimum by putting the total  
Resistance in the field circuit and the switch is closed to  
perform short circuit test. Since short circuit test is under  
understaturated condition of the machine it will be a linear  
characteristics passing through the origin and one single reading  
is enough. the two characteristics are plotted and according to  
the definition of synchronous Impedance the value is calculated from  
the graph.

method of test for subtransient Reactance. Apply voltage across  
any two terminals except the neutral with the rotor at Rest and  
short circuited on itself through an ammeter. the rotor is  
is rotated by hand and it will be observed that for a fixed

Voltage applied, the current in the field varies with the position of the Rotor. When the Rotor is in the position of maximum induced field current (the direct axis position of Rotor), one half the voltage required to circulate Rated current, is equal to the direct axis subtransient Reactance  $X_d''$  in per unit value, if the Rotor is in the position of minimum induced field current the quadrature axis subtransient Reactance  $X_q''$  is obtained.



measurement of subtransient Reactance of an alternator.

measurement of negative Sequence Reactance, the negative sequence Reactance of a machine is the impedance offered to the flow of negative sequence current.

The machine is drawn at Rated speed and a reduced voltage is applied to circulate approximately the rated current. It is to be noted here that since negative sequence current will flow in this case, there is possibility of hunting which will result in oscillation of the pointer of the ammeter. The mean reading may be taken, the negative sequence impedance is given by

$$Z_2 = \frac{V}{\sqrt{3}I}$$

## $\Rightarrow$ Fault calculation :-

Broadly speaking the faults can be classified as:

1. Shunt faults (short circuit).
2. Series faults (open conductor)

Shunt type of faults involve power conductor or conductor to ground or short circuit between conductors. When circuits are controlled by fuses or any device which does not open all three phases, one or two phases of the circuit may be opened while the other phases or phase is closed. These are called series type of faults. These faults may also occur with one or two broken conductors. Shunt faults are characterised by increase in current and fall in voltage and frequency whereas series faults are characterised by increase in voltage and frequency and fall in current in the faulted phases.

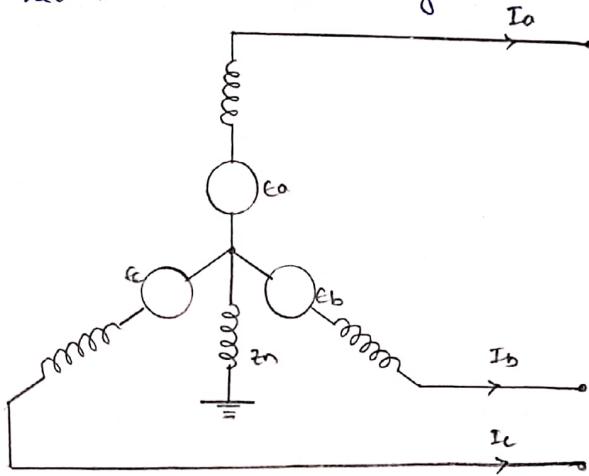
Shunt type of faults are classified as i, line-to-ground fault; ii, line-to-line faults; iii, Double line-to-ground fault; iv, 3-phase faults of these, the first three are the unsymmetrical faults as the symmetry is disturbed in one or two phases. The method of symmetrical components will be utilized to analyse the unbalance in the system. The 3-phase fault is a balanced fault which could also be analysed using symmetrical components.

The series faults are classified as i, one open conductor ii, two open conductors these faults also disturb the symmetry in one or two phases and are, therefore, unbalanced faults. The method of symmetrical components can be used for analysing such situations in the system.

Here we will discuss only the shunt type of faults.

⇒ Sequence network equations:-

These equations will be derived for an unloaded alternator with neutral solely grounded. assuming that the system is balanced the generated voltage are of equal magnitude and displaced by  $120^\circ$ . consider the diagram.



A Balanced 3-Φ system:

Since the sequence impedances as per phase are same for all three phases and we are considering entirely a balanced system the analysis will be done on single phases bases. the positive sequence component of voltage at the fault point is the positive sequence generated voltage minus the drop due to positive sequence current in positive sequence impedance (as positive sequence current does not produce drop in negative or zero sequence impedance).

$$V_{a1} = E_a - I_{a1} Z_1$$

Similarly, the negative sequence components of voltage at the fault point is the generated negative sequence voltage minus the drop due to negative sequence current in negative sequence impedance (as negative sequence current does not produce drop in positive or zero sequence impedances).

$$V_{a2} = E_{a2} - I_{a2} Z_2$$

Since the negative voltage generated is zero, therefore

$$E_{a2} = 0$$

$$V_{a2} = -I_{a2}$$

Similarly for zero sequence voltages  $E_{a0} = 0$

$$V_{a0} = V_1 - I_{a0} Z_{g0} = -3 I_{a0} Z_n - I_{a0} Z_{g0} = -I_{a0} (Z_n + 3Z_{g0})$$

where  $Z_{g0}$  is the zero sequence impedance of the three sequence network equations are, therefore.

$$V_{a1} = E_a - I_{a1} Z_1$$

$$V_{a2} = -I_{a1} Z_2$$

$$V_{a0} = -I_{a0} Z_0$$

where,  $Z_0 = Z_{g0} + 3Z_n$  and the corresponding sequence networks for the unloaded alternator  $I_{a1}$  are shown.

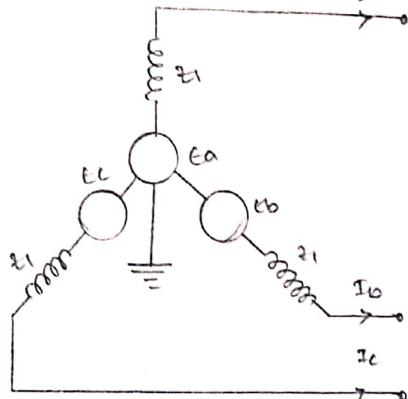


Fig: (a) (a)

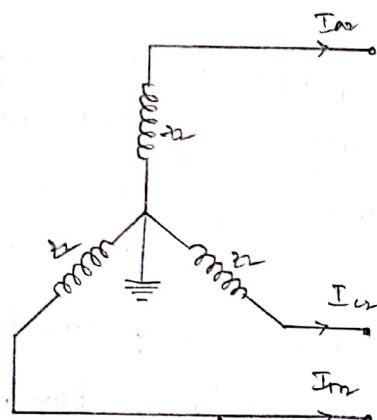
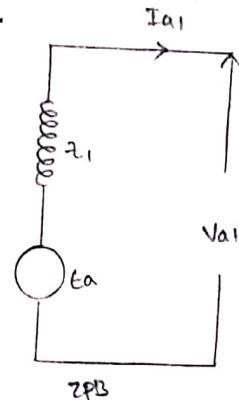


Fig: (b) (b)

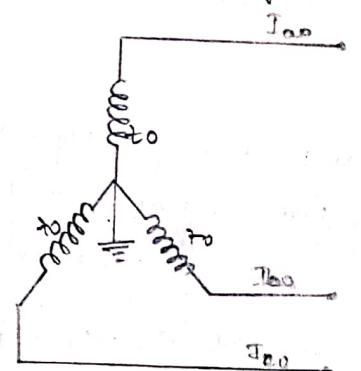
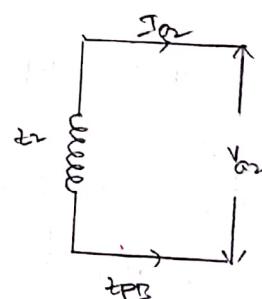
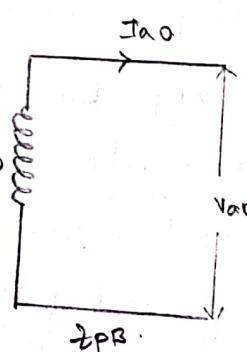


Fig: (c) (c)



(a) positive sequence network (b), negative sequence networks.

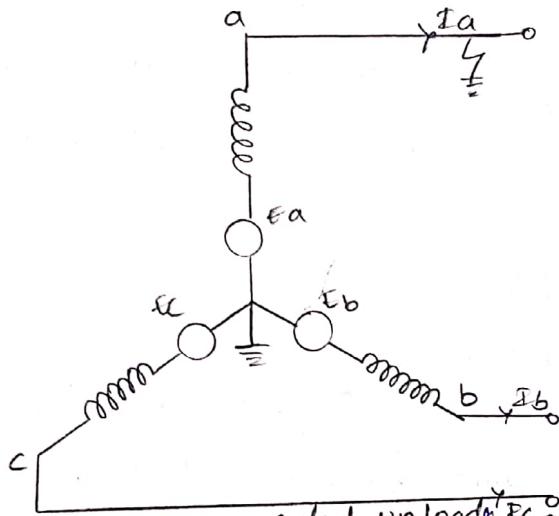
(c) zero sequence network.

~~Single~~ Single  
 $\Rightarrow$  Single line-to-ground fault :-  
 The system to be analysed let the fault take place on phase a  
 the boundary conditions are.

$$V_a = 0$$

$$\bar{I}_b = 0$$

$$\bar{I}_c = 0$$



A solidly grounded, unloaded alternator: L-G fault on phase  
 and the sequence network equations are.

$$V_{a0} = -\bar{I}_{a0} Z_0$$

$$V_{a1} = e_a - \bar{I}_{a1} Z_1$$

$$V_{a2} = -\bar{I}_{a2} Z_2$$

The solutions of these six equations will give six unknowns  
 $V_{a0}, V_{a1}, V_{a2}$  and  $\bar{I}_{a0}, \bar{I}_{a1}$ , and  $\bar{I}_{a2}$ .

$$\bar{I}_{a1} = \frac{1}{3} (\bar{I}_a + \lambda \bar{I}_b + \lambda^2 \bar{I}_c)$$

$$\bar{I}_{a2} = \frac{1}{3} (\bar{I}_a + \lambda^2 \bar{I}_b + \lambda \bar{I}_c)$$

$$\bar{I}_{a0} = \frac{1}{3} (\bar{I}_a + \bar{I}_b + \bar{I}_c)$$

Substituting the values of  $\bar{I}_b$  and  $\bar{I}_c$  from equations.

$$\bar{I}_{a1} = \bar{I}_{a2} = \bar{I}_{a0} = \bar{I}_a / 3$$

Equations can be written in terms of symmetrical components.

$$V_a = 0 = V_{a1} + V_{a2} + V_{a0}$$

now Substituting the values of  $V_{a0}, V_{a1}$  and  $V_{a2}$  from the sequence network equations.

$$E_a - I_{a1}Z_1 - I_{a2}Z_2 - I_{a0}Z_0 = 0$$

$$I_{a1} = I_{a2} = I_a$$

Since  
equation becomes.

$$E_a - I_{a1}Z_1 - I_{a1}Z_2 - I_{a1}Z_0 = 0$$

$$I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z_0}$$

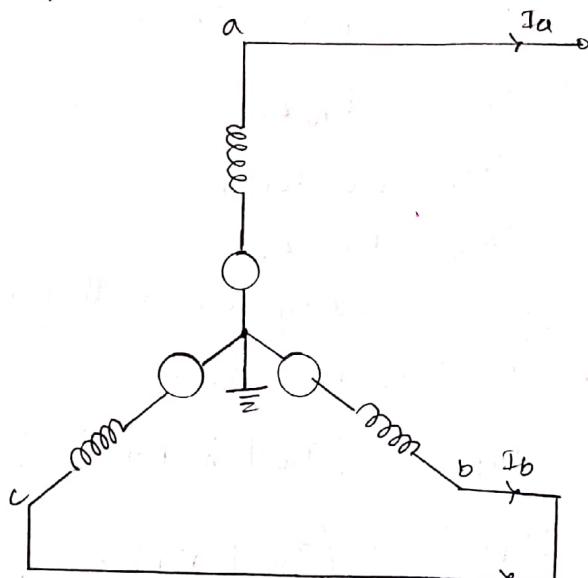
$\Rightarrow$  line-to-line fault :- the line-to-line fault takes place on phases b and c. The boundary conditions are

$$I_a = 0$$

$$I_b + I_c = 0$$

$$V_b = V_c$$

and the sequence network equations are given by the solution of these six equations will give six unknowns.



L-L fault on an unloaded and neutral grounded alternator

using the relations  $I_{a1} = \frac{1}{3} [I_a + \lambda I_b + \lambda^2 I_c]$

$$I_{a2} = \frac{1}{3} [I_a + \lambda^2 I_b + \lambda I_c]$$

$$I_{a0} = \frac{1}{3} [I_a + I_b + I_c]$$

and substituting for  $I_a$ ,  $I_b$  and  $I_c$ .

$$I_{a1} = \frac{1}{3} [0 + \lambda I_b - \lambda^2 I_b]$$

$$= \frac{1}{3} [\lambda - \lambda^2] I_b$$

$$I_{a2} = \gamma_s [0 + \lambda^2 I_b - \lambda I_b]$$

$$= \frac{I_b}{3} [\lambda^2 - \lambda]$$

$$I_{a0} = \gamma_s (0 + 0) = 0$$

which means for a line-to-line-fault the zero sequence component of current is absent and positive - sequence component of current is equal in magnitude but opposite in phase to negative - ve sequence component of current.

$$I_{a1} = -I_{a2}$$

The symmetrical L-L fault condition zero sequence network is not required and the positive and negative - sequence networks are to be connected in opposition as  $I_{a1} = -I_{a2}$ .

Now from equations.

$$V_b = V_{a0} + \lambda^2 V_{a1} + \lambda V_{a2}$$

$$V_c = V_{a0} + \lambda V_{a1} + \lambda^2 V_{a2}$$

Substituting these Relation in equations.

$$V_{a0} + \lambda^2 V_{a1} + \lambda V_{a2} = V_{a0} + \lambda V_{a1} + \lambda^2 V_{a2}$$

$$(\lambda^2 - \lambda) V_{a1} = (\lambda^2 - \lambda) V_{a2}$$

$$V_{a1} = V_{a2}$$

That is, positive - sequence component of voltage equals the negative - sequence component of voltage. This also means that the two sequence networks are connected in opposition. now making use of the sequence network equation and the equation.

$$V_{a1} = V_{a2}$$

$$E_a - I_{a1} z_1 = -I_{a2} z_2 = I_{a1} z_2$$

$$I_{a1} = \frac{E_a}{z_1 + z_2}$$

$\Rightarrow$  Double line to ground fault :-

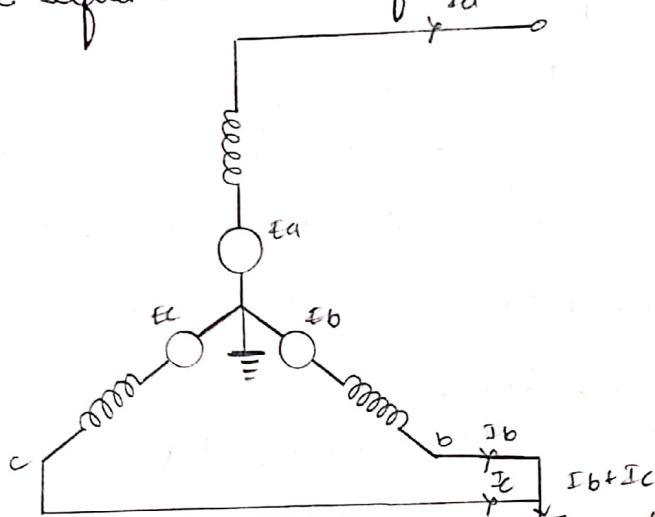
Double line to ground fault takes place on phases b and c.  
The boundary conditions are.

$$I_a = 0$$

$$V_b = 0$$

$$V_c = 0$$

and the sequence network equations are given by.



A solidly grounded, unloaded alternator,  $\frac{1}{\sqrt{3}} - L-G$  fault

The solution of these six equations will give the six unknown symmetrical components using the equations and substituting for  $V_a$ ,  $V_b$  and  $V_c$  from.

$$V_{ao} = \frac{1}{3} [V_a + V_b + V_c]$$

$$= V_a / 3$$

$$V_{a1} = \frac{1}{3} [V_a + \lambda V_b + \lambda^2 V_c]$$

$$= V_a / 3$$

$$V_{a2} = \frac{1}{3} [V_a + \lambda^2 V_b + \lambda V_c]$$

$$= V_a / 3$$

$$V_{ao} = V_{a1} = V_{a2}$$

using the relation of voltages and substituting in the sequence network equations

$$V_{ao} = V_{a1}$$

$$-I_{ao} Z_o = E_a - V_{a1} Z_1$$

$$I_{ao} = \frac{E_a - V_{a1} Z_1}{Z_o}$$

(10)

Similarly,  $V_{a2} = V_{a1}$

$$-I_{a2}Z_2 = E_a - I_{a1}Z_1$$

$$I_{a2} = -\frac{E_a - I_{a1}Z_1}{Z_2}$$

Now from equation

$$I_a + I_{a1} + I_{a2} + I_{ao} = 0$$

Substituting values of  $I_{a2}$  and  $I_{ao}$  from equations.

$$I_{a1} = -\frac{E_a - I_{a1}Z_1}{Z_2} - \frac{E_a - I_{a1}Z_2}{Z_0} = 0$$

Rearranging the terms gives.

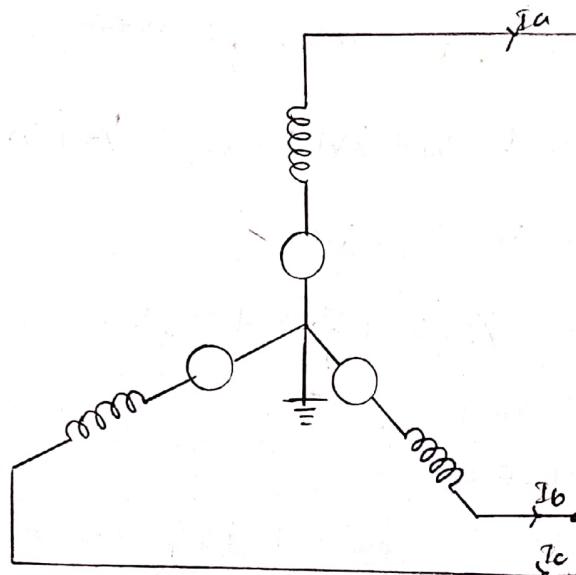
$$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}}$$

$\Rightarrow$  3-phase fault:-

The boundary conditions are

$$I_a + I_b + I_c = 0$$

$$V_a = V_b = V_c$$



A 3-φ neutral grounded and unloaded  
alternator 3-φ shorted.

Since  $|I_a| = |I_b| = |I_c|$  and if  $I_a$  is taken as

Reference.

$$I_b = \lambda^2 I_a \text{ and } I_c = \lambda I_a.$$

using the Relations

$$I_{a1} = \frac{1}{3} [I_a + \lambda I_b + \lambda^2 I_c]$$

and substituting the values of  $I_b$  and  $I_c$ .

$$I_{a1} = \frac{1}{3} [I_a + \lambda^3 I_a + \lambda^3 I_a]$$

$$= I_a$$

$$I_{a2} = \frac{1}{3} [I_a + \lambda^2 I_b + \lambda I_c]$$

Substituting for  $I_b$  and  $I_c$  in terms of  $I_a$ .

$$I_{a2} = \frac{1}{3} (I_a + \lambda^4 I_a + \lambda^2 I_a)$$

$$= \frac{1}{3} (I_a + \lambda I_a + \lambda^2 I_a)$$

$$= \frac{I_a}{3} (1 + \lambda + \lambda^2)$$

$$= 0$$

$$I_{a0} = \frac{1}{3} (I_a + I_b + I_c)$$

which means that for a 3-phase fault zero as well as negative-sequence components of current are absent and the positive-sequence component of current is equal to the phase current.

Now using the voltage boundary Relation.

$$V_{a1} = \frac{1}{3} (V_a + \lambda V_b + \lambda^2 V_c) = \frac{1}{3} (V_a + \lambda V_d + \lambda^2 V_a)$$

$$= V_a \frac{1}{3} (1 + \lambda + \lambda^2) = 0$$

$$V_{a2} = \frac{1}{3} (V_a + \lambda^2 V_b + \lambda V_c)$$

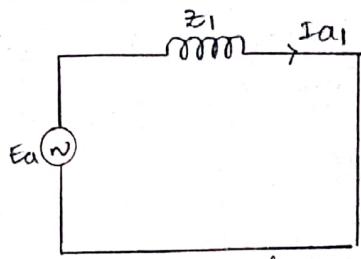
$$= 0$$

$$V_{a0} = 0$$

$$V_{a1} = 0 = E_a - I_{a1} Z_1$$

$$I_{a1} = \frac{E_a}{Z_1}$$

The sequence network is shown in.



Interconnection of sequence network

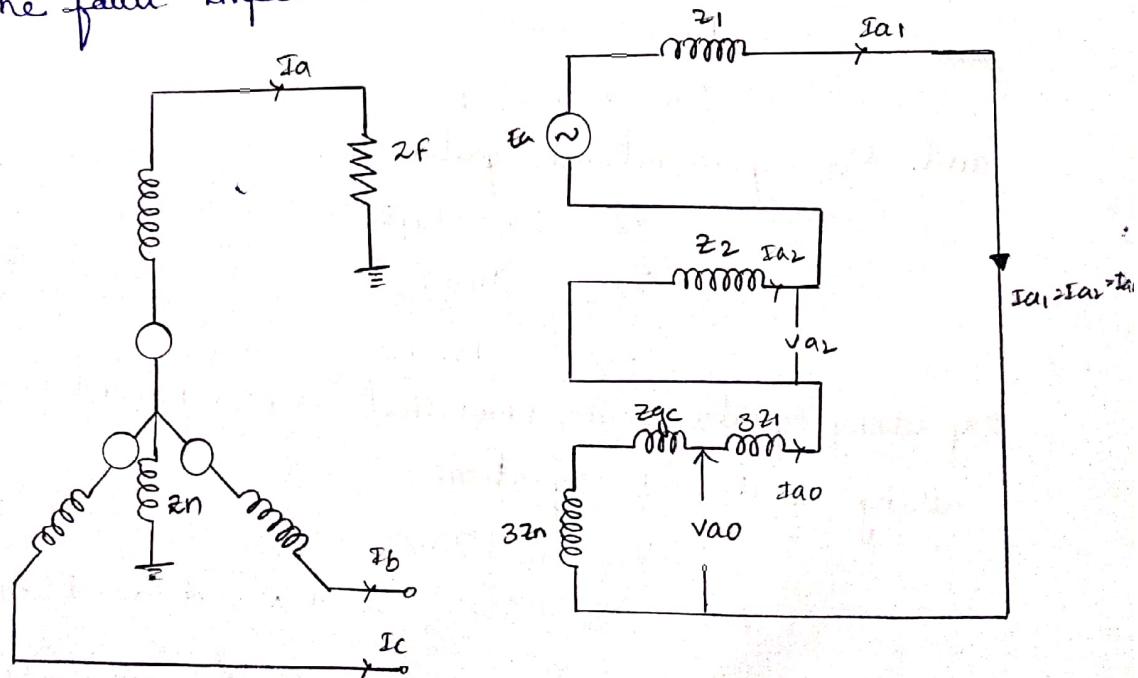
 $\gamma-\phi$  fault

From the analyses of the various faults, the following observations are made :-

1. Positive sequence currents are present in all unsymmetrical faults.
2. Negative sequence currents are present when the neutral of the system is grounded and  $\gamma < \alpha$
3. Zero sequence currents are present when the neutral of the system is grounded and the fault also involves the ground, and magnitude of the neutral current is equal to  $3I_{ao}$ .

$\Rightarrow$  Line-to-ground fault with  $Z_f$  :-

The fault impedance is  $Z_f$  and the neutral impedance  $Z_n$ .



The boundary conditions are.

$$V_a = I_a Z_f$$

$$I_b = 0, I_c = 0$$

$$V_{a0} = -I_{a0}(Z_{g0} + 3Z_n)$$

$$V_{a1} = E_0 - I_{a1}Z_1, V_{a2} = -I_{a2}Z_2$$

The solution of these equations gives the unknown quantities, from equations and the boundary conditions above.

$$I_{a1} = I_{a2} = \bar{I}_{a0} = I_a/3$$

$$V_{a1} + V_{a2} + V_{a0} = V_a = 3I_{a1}(Z_f)$$

$$E_a - I_{a1}Z_1 - I_{a2}Z_2 - I_{a1}(Z_0 - 3Z_n) = 3I_{a1}(Z_f)$$

$$E_a = I_a, [Z_1 + Z_2 + \{(Z_0 + 3Z_n) + 3Z_f\}]$$

$$I_{a1} = \frac{E_0}{Z_1 + Z_2 + (Z_0 + 3Z_n) + 3Z_f}$$

Since  $I_{a1}, I_{a2}$  and  $I_{a0}$  are known,  $V_{a1}, V_{a2}$  and  $V_{a0}$  can be calculated from the sequence network equations. The sequence network interconnection.

$\Rightarrow$  Line-to-line fault with  $Z_f$ :

The boundary conditions.

$$I_a = 0$$

$$I_b + I_c = 0$$

$$V_b = V_c + I_b Z_f$$

and the sequence network equations are.

$$V_{a1} = E_a - I_{a1}Z_1$$

$$V_{a2} = -I_{a2}Z_2$$

$$V_{a0} = -I_{a0}Z_0$$

By using equations, we know that  $I_{a1} = -I_{a2}$  and  $I_{a0} = 0$  using equations in equation.

$$V_b = V_c + I_b Z_f$$

$$V_{a0} + \lambda^2 V_{a1} + \lambda V_{a2} = V_{a0} + \lambda V_{a1} + \lambda^2 V_{a2} + (\lambda^2 I_{a1} + \lambda I_{a2}) Z_f$$

$$\lambda^2 V_{a1} - \lambda V_{a1} = (\lambda^2 - \lambda) V_{a2} + (\lambda^2 I_{a1} - \lambda I_{a1}) Z_f$$

$$V_{a1} = V_{a2} + I_{a1} Z_f$$

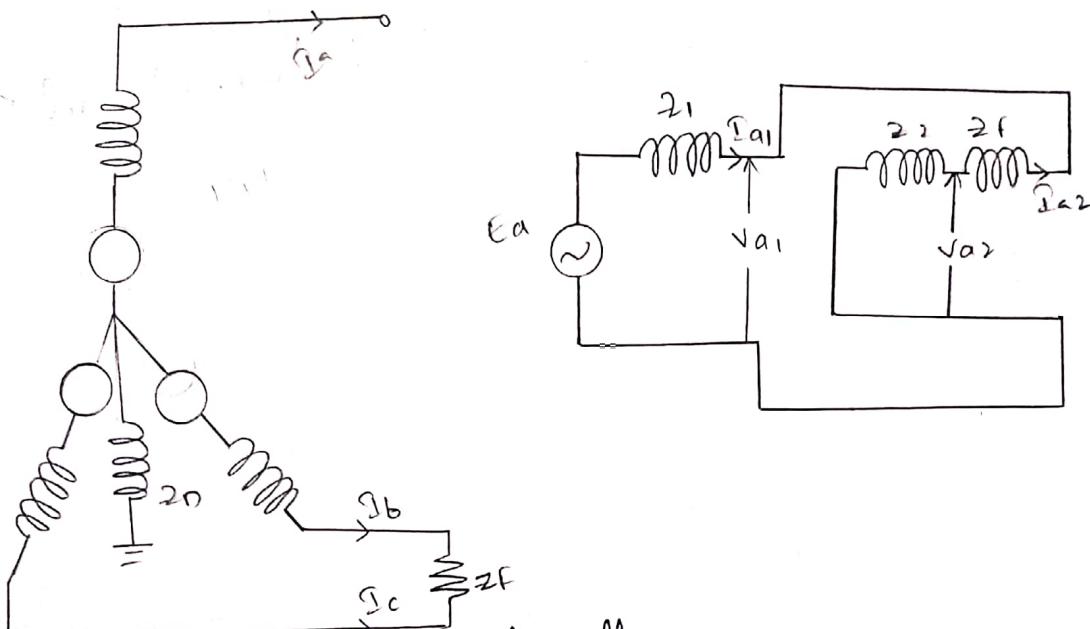
Now substituting for  $v_{a1}$  and  $v_{a2}$  from the sequence network equations

$$E_a - I_{a1} Z_1 = -I_{a2} Z_2 + I_{a1} Z_f$$

$$E_a - I_{a1} Z_1 = I_{a1} (Z_2 + Z_f)$$

$$I_{a1} = \frac{E_a}{Z_1 + (Z_2 + Z_f)}$$

The outer connection of the sequence network.



$\Rightarrow$  Double line-to-ground fault :-

boundary fault impedance is  $Z_f$  and neutral impedance  $Z_n$ . The boundary fault conditions are

$$I_b = 0$$

$$V_b = V_c = (I_b + I_c) Z_f$$

and the sequence network equations are.

$$V_{a1} = E_a - I_{a1} Z_1$$

$$V_{a2} = -I_{a1} Z_2$$

$$V_{ao} = -I_{ao} (Z_0 + 3Z_f)$$

we know that  $(I_b + I_c) = 3I_{ao}$ .

$$V_b = V_c = 3I_{ao} Z_f$$

$$\lambda^2 V_{a1} + \lambda V_{a2} + V_{ao} = \lambda V_{a1} + \lambda^2 V_{a1} + V_{ao}$$

$$V_{a1} = V_{a2}$$

using the relation in equation.

$$V_b = 3I_{ao} Z_f$$

$$\lambda^2 V_{a1} + \lambda V_{a1} + V_{ao} = 3I_{ao} Z_f$$

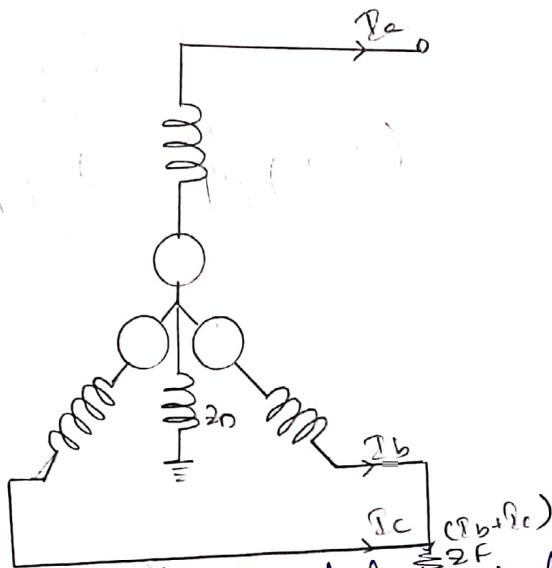
$$-V_{a1} + V_{ao} = 3I_{ao} Z_f$$

$$V_{a1} = V_{ao} = 3I_{ao} Z_f.$$

Substituting for  $V_{a1}$  and  $V_{ao}$  from the sequence equation and expressing  $I_{ao}$  in terms of  $I_{a1}$ , we get.

$$E_a - I_{a1} Z_1 = -I_{ao} (Z_0 + 3Z_n) - 3I_{ao} Z_f.$$

$$I_{ao} = -\frac{E_a - I_{a1} Z_1}{Z_0 + 3Z_n + 3Z_f}$$



Before we proceed further to study the faults on an actual power system where the alternator may be connected to a transmission line through a transformer or any other interconnected system we will like to study the sequence network representation of various components like a generator, transformer, a synchronous motor etc.

$\Rightarrow$  Sequence networks:-

The positive sequence network is in all respects identical with the usual networks considered each synchronous machine must be considered as a source of emf, which may vary in magnitude and phase position depending upon the position of power and reactive volt amper just prior to the occurrence of the fault. The positive sequence voltage at the point of fault will drop, the amount being dependent upon the type of fault; for 3-phase faults it will be zero; for double of faults; for 3-phase faults, it will be zero; for double line-to-ground faults, line-to-line fault and single line-to-ground fault, it will

be higher on the order stated.

The negative sequence network is in general quite similar to the positive sequence network except for the fact that since no negative sequence voltage are generated, the source of emf is absent.

The zero sequence network is in general quite similar to the positive sequence network like we will be free of external voltages, the flow of current resulting from the voltage at the point of fault. The impedance to zero sequence current are very frequently different from the positive or negative sequence currents. The transformer and generator impedance will depend upon the type of connections whether star or delta connected; if star, whether grounded or not.

### \* Reactors:-

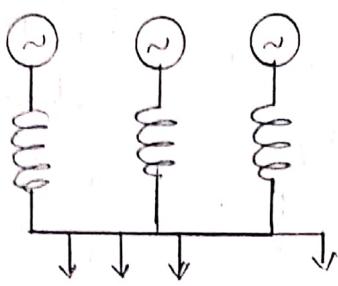
Reactor is a coil which has high inductive reactance as compared to its resistance and is used to limit the short circuit current during fault conditions. To perform this function it is essential that magnet saturation at high current does not reduce the coil reactance. If an iron cored inductor is expected to maintain constant reactance for currents two to three times its normal value it will turn out to be very costly and heavy. Therefore air cored coils having constant inductance are generally used for current limiting.

### Reactors.

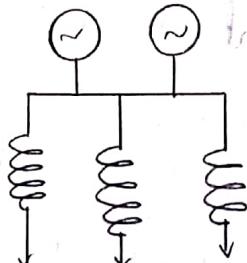
i) generator Reactors:- The reactance of modern alternators may be as high as 2.0 p.u which means even a dead short at the terminals of the alternator will result in a current less than full load current, and therefore, no external reactor is required for limiting the short circuit current of such a machine. However, if some old machines are being

used along with the modern alternator, these old machines need the Reactors for limiting the short circuit current. The location of Reactors is given.

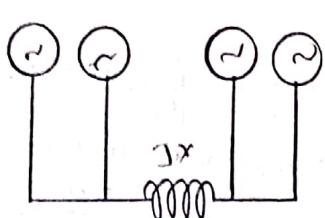
ii) Feeder Reactor: - The per unit value of Resistance of a feeder based on its Rating may be small but when compared with the Rating of whole system, its value is quite large and hence, a small Reactor will be effective in limiting the short circuit current. Should a fault occur close to the generating stations. In case the feeder Reactor is not there, a fault in such a location would bring the bus bar voltage almost down to zero value. And there is a possibility of various generators falling out of step, we know that, to improve the transient stability of a system the critical clearing angle should be as small as possible. The breakers should be as fast as possible, in order to obtain this situation and at the same time to reduce the current to be interrupted the feeder must be associated with a Reactor.



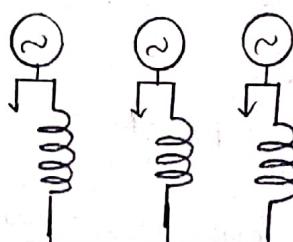
(a)



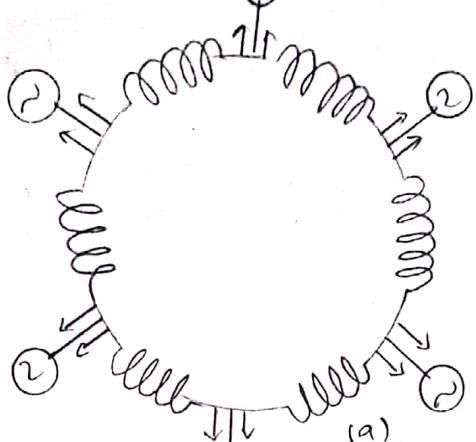
(b)



(c)



(d)



iii, Busbar Reactor:- There are three methods of interconnecting the busbar through the reactors as the simple method is suitable for plants of moderate output whereas for large sized plants either the star or ring system of connection is used. It is to be noted that any transfer a power from say section A to section B of the generators, a difference in potential between the bus section is developed.